Conditions for the Ontic Treatment of Mathematics: Einstein's Separability Principle and Material-ladenness

A striking feature of Einstein's separability principle, a principle that figured centrally in his criticism against quantum theory, is that it is based on the ontic treatment of the elements of field theories. It is the central aim of this work to suggest and motivate an answer to the question: *"What are the conditions that allow for the ontic treatment of mathematical concepts by epistemic agents?"* by developing an account for the specific case of Einstein and the separability principle. My answer to the general question will be that one such condition is the "material-ladenness" of mathematical concepts upon which such a treatment is conditioned, however in a contingent way.

In particular, I will first introduce the separability principle and the treatment of the mathematical formalism of field theories that underlies Einstein's use of the former (Sec.1). I then introduce the historiographic framework of mental models and discuss the "material-ladenness" of mental structures including mathematical concepts, which is captured quite naturally by this framework (Sec.2). This I use in order to suggest an explanation for the fact that Einstein was able to employ the mathematical elements of field theory to support his argument against quantum theory by means of the separability principle (Sec.3,4). This finding suggests material-ladenness as an adequate way for understanding the conditions of an ontic treatment of mathematical concepts, and mental structures more generally (Sec.5). Finally, characteristics of the presented approach are clarified by relating it to two relevant approaches in the anglophone historiographic community (Sec.6).

1 Einstein's separability principle

In his (1985), Don Howard argues convincingly that Einstein's rejection of the formalism of quantum mechanics as incomplete stemmed from his firm belief in a metaphysical principle, which Howard calls the separability principle. Howard sketches the development and gradual demarcation of this principle in Einstein's thinking from its earliest documentation in a letter to Schrödinger from 1935 (following the publication of the seminal EPR-paper) to its clearest statement in a Dialectica-paper in 1964, where Einstein (1948, 321f.) says that

> it is characteristic of [real] physical things that they are conceived of

> as being arranged in a space-time continuum. Further, it appears to be essential for this arrangement of the things introduced in physics that, at a specific time, these things claim an existence independent of one another, insofar as these things 'lie in different parts of space'.

Thus, in Howard's (1985, 173) words, the separability principle asserts that "any two spatially separated systems possess their own separate real states". Further, Einstein remarks that

> Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space-elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them (Einstein, 1948, 321, transl. taken from (Howard,1985,188)).

Howard (1989) investigates this connection between Einstein's concept of a field and separability. He argues that the separability principle as understood by Einstein in the above sense, can fulfil its purpose, i.e. the coordination between a system's spatio-temporal location and its ontological state, only if the mathematical field concept is employed ontologically: A field mathematically needs to be well-defined for every point of a manifold. For Einstein, the manifold-points, whose values define the field, are not just a mathematically convenient tool but are understood as providing the means to individuate systems as "portions of reality".

Underlying this analysis, in Howard's opinion, is a fundamentally atomistic and reductionist understanding of field theories: He sees in "field theories the inevitable culmination of the inherent logic of atomism; they represent atomism carried to its logical extreme-a sea of infinitesimal atoms, any two atoms having between them a continuum of other atoms" (ibid., 244). Only by assigning this ontological role to the mathematical formalism of field theories, then, can Einstein employ the separability principle against quantum theory.

It is this act of assignment from where my investigation vantages and that it returns to. It highlights an obvious question: *What was it that allowed for the ontic treatment of the mathematical field theoretic formalism by Einstein?* This question is justified simply because there is no obvious reason why such a treatment should be possible per se.

2 The mental models framework

Mental Models

The answer I give requires a suitable framework: Mental models.[1] A mental model is "an internal knowledge representation structure serving to simulate or anticipate the behaviour of objects or processes. It possesses "terminals"

[1] A large number of different approaches that make use of the term "mental model " are around. Here I focus explicitly on the the one advocated by the J.Renn's department in the Max Planck Institute for History of Science.

or "slots" that can be filled with empirically gained information, but also with default assumptions resulting from prior experience." (Renn and Sauer, 2007, 127).

Two features of the mental models approach are particularly relevant for our purposes: Firstly, inputs of mental models can be changed successively and in a one-by-one fashion, subject only to the stability of the internal architecture of the system of knowledge, in the constitution of which the mental model figures. This stability is dependent on the degree of correlation between different mental models. Using this property, we can therefore "describe and explain modes of inference specific to some object of investigation and the historical change of these modes that used to be investigated by traditional philology and metaphysical logic."(Renn and Damerow, 2006, 7)

Material-ladenness

Secondly, mental models capture the intimate connection between the mental and physical realm in the role they play for the dynamics of knowledge representation and formation: Mental models and the cognitive structures they represent, are inherently "material-laden". To see this, we need to look at the way that mental models are constructed:

Mental models can be externally represented by material systems with which the same (possibly purely symbolic) actions can be performed as with the real objects that the mental model represents (Damerow, 2007, 25). Now,

> unlike in the case of their simple application to directly control real actions, symbolic actions in their role of supporting the construction of mental models can completely substitute the real actions, because they share essential physical qualities with the objects and actions for which they stand. Symbolic actions in the system of rules of a first-order representation thus initiate the construction of the same mental model as actions with the real objects they represent (ibid.).

In the case of Damerow's study of the mental models logico-mathematical thought, concepts then

> are abstracted not directly from the objects of cognition, but from the coordination of the actions that they are applied to and by which they are somehow transformed. According to this assumption the emergence of mental operations of logico-mathematical thought is based on the internalisation of systems of real actions. The internalised actions are the starting-point for meta-cognitive constructions. (ibid.,22)

It is through the construction of novel mental models as processes of *formation by internalisation* of the actions that represent existing mental models by means of external, material representations that we can understand how theoretical concepts even of what is often regarded as the most "abstract" epistemic discpline of all, mathematics, carry with them traces of the physical world. The property of concepts that results from this process and that is captured by corresponding mental models, I call "material-ladenness".[2]

Material-ladenness can take different forms, corresponding to different ways in which the external world can constrain possible actions on the external representations. Thus, both the physical environment and the laws that it is subjected to as well as culture-specific material cultures, in which the mental model is constructed, result in material-ladenness. For Einstein's case, I concentrate on the former of these points, but I will discuss the latter in Sec.6.

By only taking these two properties together, mental models already provide a rich and flexible account of the dynamics of the translation and construction of knowledge structures:

> [Mental models] bridge several levels of knowledge that represent the same object in different forms of knowledge, from the level of practical knowledge up to the level of scientist's theories. In particular, with them implicit inferences can be captured, that are embodied in the practical logic and do not possess an explicit representation in spoken or written form.[3] Renn and Damerow (2006, 6, trans. by myself)

Moreover, the architecture of mental models allows one to reconstruct a "biography" of the theoretical structure represented. Following the dynamics of the development of mental models presented above these mental models are not substituted as a whole in the course of scientific reflection: Instead, their original architecture is possibly replaced in a piece-meal fashion. Consequently, the mental activities that are performable in a sophisticated mental model such as that of the Lorentz model, can retain constraints of less sophisticated mental models of "intuitive mechanics"[4] and classical physics. Such a biography is exactly what I will try to develop with Einstein's field concept.

3 Einstein's mental model

Now that we are equipped with the necessary tools, let me present my answer to the question above: It is the material-ladenness of the mental model of a field,

[2] This term is obviously chosen in analogy with "theory-ladenness": Theory-ladenness undermines our hope to understand the world objectively, where the latter means something like "the stuff that it is"; Similarly, material-ladenness undermines our hope to ever successfully think something objectively, where the latter means something like "the pure mental essence remaining after successful and complete abstraction."

[3] Mentale Modelle überbrücken verschiedene Ebenen des Wissens, die denselben Gegenstand in unterschiedlichen Wissensformen repräsentieren, von der Ebene des Handlungswissens von Praktikern bis zur Ebene der Theorien von Wissenschaftlern. Mit ihnen lassen sich daher insbesondere auch implizite Schlussfolgerungen erfassen, die in der Handlungslogik von Praktikern verkörpert sind und keine explizite Darstellung in sprachlicher oder schriftlicher Form besitzen.

[4] "Intuitive physics is based on experiences acquired almost universally in any culture by human activities. Experiences relevant to intuitive mechanical knowledge include, for instance, the perception of material bodies and their relative permanence, their impenetrability, their mechanical qualities, and their physical behaviour." (Renn et al., 2003, 45)

that allowed for Einstein's ontic treatment of the manifold-points. To see this, I will first translate Einstein's field concept into an appropriate mental model and then investigate the material-ladenness of this model. Howard's work itself does not help us with this task: No structures resembling mental models are present in his work.[5]

The Lorentz model

In a detailed study, Renn and Sauer (2007) elaborate the mental model that served as Einstein's main heuristic and formal guiding device in his search for a general theory of relativity. This mental model essentially represents Lorentzian field theory and is therefore simply called the "Lorentz model". As the authors show, the particular architecture of this model can be employed to understand how Einstein could arrive at a theory that overthrew classical concepts with the means provided by the latter while still retaining a striking formal similarity to it (ibid.,119) in just the way presented above.

Crucially for us, the Lorentz model

> results from the integration of mental models referring to two kinds of physical substances, the model of an extended, space-filling physical medium traditionally labelled as "aether" and the model of matter constituted by particles. (ibid.,139)

These two models are responsible for the well-known conceptual inconsistency of Lorentzian Electrodynamics, where point-like carriers of charge are treated ontologically on a par with extended fields Frisch (2005, Pt.1). Mathematically this manifests itself in the fact that the theory requires both the equations of motion for point-like particles and the field equation for the field because one cannot be produced from the other dynamically Renn and Sauer (2007, 137).

Importantly, although Einstein has been very much aware of this difficulty[6] and even traces its origin back to the mechanisms of abstraction,[7] he admits that both the special and the general theory of relativity suffer from it.[8]

[5] Instead, Howard (1985, 192) believes the reason for Einstein's ontic treatment to have been a kind of conscious choice due to lack of a better principle of individuation. This seems rather artificial to me.

[6] see (Einstein, 1950, 14),(Einstein, 969a, 27,36f.),(Einstein, 969b, 675f.)

[7] "The material point is our only mode of representing reality when dealing with changes taking place in it, the solitary representative of the real, in so far as the real is capable of change. Perceptible bodies are obviously responsible for the concept of the material point; people conceived it as an analogue of mobile bodies, stripping these of the characteristics of extension, form, orientation in space, and all "inward" qualities, leaving only inertia and translation and adding the concept of force. The material bodies, which had led psychologically to our formation of the concept of the "material point," had now themselves to be regarded as systems of material points." (Einstein, 1954, 267)

[8] "The special and general theories of relativity [...] have so far been unable to avoid the independent introduction of material points and total differential equations." (Einstein, 1954, 269)

The "biography" of the Lorentz model

We can recognise the material-ladenness of the Lorentz model by understanding that of the mental model of matter constituted by particles: The architecture of this mental model that determines the actions we can mentally perform with particles in a field is governed by the mental model of the impenetrability of matter and that of object permanence. The former, impenetrability of matter, results from the individual experience of the inability of penetrating solid objects and the latter, object permanence, results from the experience that objects remain existent even after they left the visual, or sensual, field. Object permanence, together with the mental model that represents it, is among the most prominent results of Piaget's work on the early cognitive development of a children (Piaget, 1950, in particular Vol.I, Ch.3) Hence, both of these mental models result from "human activities related to our physical environment" and their development "is built up in ontogenesis" (MPIWG, 2001).

4 (Material) reality in Einstein's thinking

We are now, at last, in a position where we can use what we have heard above in order to relate material-ladenness to Einstein's incompleteness argument:

Einstein's formulation of the separability principle requires, as we have heard from Howard, that the manifold points are interpreted ontologically. The possibility of such an interpretation can be explained by the capacity of mental models to show how mental concepts - such as the mathematics Einstein made use of - can develop to a high degree of sophistication and complexity while still being fundamentally governed by the architecture of older, replaced mental models. In Einstein's case, the material-ladenness of the mental model of an impenetrable and permanent mass particle has found its way into the mental model of a field. This allowed Einstein to think of the mathematical concept of the points of the manifold defining a field as carrying its own and independent reality each, simply because each of them could be subjected to the same actions mentally as a simple, permanent and impenetrable object in the macroscopic physical world, the independent existence of which Einstein never had reason to doubt. Subsequently, Einstein could employ this quality when articulating his criticism against quantum mechanics.

This interpretation of the underlying reasons for the possibility of Einstein's ontic treatment of the manifold that I developed above is supported by the following two points:

Abstracting space

Firstly, in the opening of his 1933 Herbert Spencer lecture, Einstein says that

> [T]o the discoverer in [theoretical physics], the constructions of his imagination appear so necessary and so natural that he is apt to treat them not as the creations of his thoughts but as given realities. Einstein (1934b, 163)

While, of course, no mention of any kind of material-ladenness as the source of this quality of the constructions is made here, Einstein admits here of the "reality" of the theoretical concepts in the physicist's mind. More relevant for my argument is the following, extensively quoted, passage from Einstein's "Das Raum-, Äther- und Feld-Problem der Physik" from 1934, published only one year before the publication of the EPR-paper and the earliest documentation of Einstein's attempts to articulate what was later to become his separability principle:

> [The concept of space] seems to presuppose the concept of the solid body. [...] The correspondence between certain visual and tactile impressions, the fact that they can be continuously followed through time, and that the impressions can be repeated at any moment (touch, sight), are some of those characteristics. Once the concept of the solid body is formed in connection with the experiences just mentioned-which concept by no means presupposes that of space or spatial relation - the desire to get an intellectual grasp of the relations of such solid bodies is bound to give rise to concepts which correspond to their spatial relations. Einstein (1934a, 278)

Einstein's sketch of the process of abstraction of the concept of space resembles astonishingly the same process as described by the mental model framework. It is, moreover, easy to see the connection to the mental models of object permanence and the impenetrability of matter in the above passage. Further more, following the quote in footnote 7, we can see how the treatment of the *mathematical* manifol-point is rendered possible by the mental model of the *material* point. If we assume that these words of Einstein are, even if not motivated by his own experience, at least not inconsistent with it, they support my claim.

The hole argument

Secondly, Einstein's ontological treatment of an element of mathematics in the case of field theory, so faithfully documented by Howard following the publication of the EPR paper in 1935, is not the only case in point. As Norton (2005) argues, what underlies Einstein's construal of the notorious and erroneous hole argument[9] 1914 in the course of Einstein's search for the general theory of relativity, was the latter's treatment of coordinate points in the Newtonian limit as physically real. If this is true, and Norton's argument is motivated by the Zurich Notebook, the ontological (mal-)treatment of mathematics by Einstein occurs already much earlier than 1930.

This is important in so far as it precedes Einstein's turn from Machian positivism to the belief that "the actual creative principle lies in mathematics Einstein (1934b, 164).[10] Instead, Einstein's inattention to the presumptions on

[9] The hole argument attemps a refutation of spacetime substantivalism. It was conceived by Einstein to test the general covariance properties of the metric and coordinate systems of candidates for a general theory of relativity. See (Norton, 2011)

[10] for a discussion, see (Holton, 1968; Howard, 2004)

which the hole argument depends, emphasised by Norton (2005, 87), can be taken to reflect their apparent "naturalness" to Einstein.

While not directly supporting my argument, together these circumstances at least make attempts to argue for some kind of metaphysical platonism as the actual reason underlying Einstein's treatment of mathematics less convincing. Admittedly, the case of the hole argument is sufficiently different from the separability principle to make a separate investigation of the material-ladenness in play necessary in order to serve my argument in the above way.

The sceptical counter

I should stress at this point this work's concern exclusively with the modality of ontic treatment and not with the reason for such a treatment itself. This is reflected in the question above: I do not ask: What made Einstein treat the manifold ontologically?[11] For this reason, sceptical counter-arguments founding on the fact that many of Einstein's contemporaries, themselves subject even to the same political and cultural background, could employ field theoretic concepts without running into ontological problems, miss the point. It certainly is true that the statistical interpretation of Schrödinger's wave mechanics, first suggested by Born, employs the above field concept without ontological commitment, for example by the latter (Born, 1971, 209). But this is not my concern here.

5 Conditions for an ontic treatment

In the end of Section 1 I asked: *What was it that allowed for the ontic treatment of the mathematical field theoretic formalism by Einstein?* My answer to this question was developed over sections 2-4. The above question, however, is only a special case of the more general question: *What are the conditions that allow for the ontic treatment of mathematical concepts by epistemic agents?* In this section, I argue for the usefulness of the approach presented in Einstein's case as a (partial) answer as well to this latter question: The employment of mathematical structures in our thinking, or epistemic activities in a broader sense, is accompanied by traces of the physical environment from which these structures have been abstracted in the first place. It is the traces that make it possible for us to treat these structures ontically. And, of course, the traces above are just the material-ladenness I am on about.

[11] For an account on this question, see (Renn, 2004; Renn and Sauer, 2007), where a "double strategy" is suggested as the growth beed for Einstein's successful research on GTR. In it, the heuristic constraints set by established physical principles and the formal-logical constraints set by the mathematics are employed very closely. A testimony of its effects is given, for example, in the opening works to his ... already in 1914: During the research on GTR "a kaleidoscopic mixture of postulates from physics and mathematics has been introduced and used as heuristical tools" (ibid., 257). This mixture may have been a crucial stimulus for Einstein's ontic treatment of the mathematical formalism.

Alternatively, a more historical account for possible leanings of Einstein towards metaphysical notions is presented by (Holton, 1998).

The assumption that underlies this suggestion is that, in order to be able to interpret a mental structure ontically, it needs to possess a quality that relates it to the physical world. From the perspective of this assumption, the ideal of complete abstraction, in the original sense of the word, entails the impossibility of ontic interpretation. The material-ladenness of mental models captures this relationship between the mental structures and the physical world in a convincing manner by conserving (amongst others) spatio-temporal properties as they are experienced by epistemic agents. If this is correct, we can see how material-ladenness of mathematical structures, and thus the physical and cultural environment it reflects, conditions our ontic treatment of these mathematical structures by both rendering possible and constraining the kinds of actions we can perform with them.

While these considerations apply to mental concepts in general, my questions concern only mathematics. The reason for this is, on the one hand, Einstein's case. On the other hand, the material-ladenness of mathematics is particularly interesting for an integrated HPS, since the promotion of the epistemological rank of mathematics is generally regarded as a key step for the development of the science of the last five hundred years or so: Mathematics is assumed to represent the very essence of successful abstraction and material-ladenness undermines this assumption.[12]

But what exactly are the strengths of the material-laden mental models framework?[13] The following section critically compares the approach presented here with the work of Hasok Chang and Peter Galison. Quite naturally, this comparison also nurtures further insight into the framework's implications, allowing us to feed two birds with one scone.

6 Critical comparison

Chang

In a number of recent papers, Chang (2008; 2009; 2011) attempts to re-approach an integrated HPS by focussing on activities. One particular feature of his approach is the treatment of metaphysical or ontological principles as a contingent necessity: Epistemic agents require them only for specific activities. Thus, counting for example requires the metaphysical assumption of discreteness, but this assumption does not necessitate the promotion of discreteness to a universal and transcendental function of thinking, as it would have for Kant (Chang, 2008).
The framework presented here does not stand in opposition to Chang's approach. In fact, it relates to the focus both on activity and contingency. If our

[12] It should be emphasised that my argument is not pro some kind of social constructivism and contra metaphysical platonism: It provides an account of what makes platonism possible, not whether the latter's convictions are true or false. If anything, it undermines the antipodicity of the two positions.

[13] It should be noticed that the materiality I ascribe to the mental models is not canonical and has been contested by proponents of mental models in the course of my research.

mental models reflect the physical constraints of what can be done with external objects, then our thinking inherently incorporates agency relations, literally as a form of embodied knowledge.

Concerning its contingency, there is a strong overlap at least in the motivations underlying Chang's approach and the one of this paper: In Kant's transcendental aesthetic space and time, as the pure forms of intuition, are a priori conditions for the possibility of cognition. While the mental models approach does as well incorporate the necessity of *some* spatial or temporal moment of our mental concepts at the basis of cognition, it does not motivate this necessity by means of *demonstration*, as a kind of conceptual analysis as it is done in the transcedental aesthetic. Instead it is motivated on weaker groounds, namely as the result of *practice-related structures*, the mental agent's being-in-the-*material*-world.

As such, it leaves room for the possibility of the development of different kinds of spatial or temporal mental models in different socio-temporal and even individual contexts (see the teaching aspect below). Counters that aim at identifying Kant's "space" or "time" with the "common denominator" of all these possible spatial mental models, their "a priori nucleus", further run danger of using the mental models against its nature, due to a certain moment of irreducibility present in mental models.

We see then how mental models constrain, shape and condition our thinking and reasoning but their own architecture is contingent: It results from the complex overlap of cultural and cognitive factors on a long and short term scale, mechanisms of inheritance and is fundamentally shaped by material-ladenness.[14]

As such, the mental models framework (as interpreted here) aligns itself with Chang (2008) as an alternative to Friedman's relativised a priori by questioning the latter's assertion of the necessity of one set of universal, trans-historical constitutive principles. However, unlike for Chang, this is not because we decide, out of some methodological or epistemological considerations, to pragmatically choose different sets of coordinative principles for different tasks. Instead, it is because our cognitive structures, our "thinking flow", is the product of a

[14] This contingency is, interestingly, also recognised by Einstein, although limited to the primacy of sensual experience for our cognitive concepts: Immediately before the extensive quote above, Einstein says that

> concepts have reference to sensible experience, but they are never, in a logical sense, deducible from them. For this reason I have never been able to understand the quest of the a priori in the Kantian sense. In any ontological question, our concern can only be to seek out those characteristics in the complex of sense experiences to which the concepts refer. (Einstein, 1934a, 278)

and following it, in the context of the independence of spatial intervals from the specific solid body that can serve to fill them:

> It is evident that this independence, which is a principal condition of the usefulness of framing purely geometrical concepts, is not necessary a priori. In my opinion, this concept of the interval, detached as it is from the selection of any special body to occupy it, is the starting point of the whole concept of space. (ibid.)

complex onto- and phylogenetic environment, from which no "pure forms of intuition" or universally stable fixed points of "thinking flow" can be distilled. There is no conscious choice of these things involved, in fact the construal of the latter appears more as a kind of a posteriori justificationist activity, which may or may not helpful but in either way cannot aspire to capture "the way things go."

Galison

While Chang's work and mine share a primary concern for activities in the mental realm, the connection between the practical concern of Galison's work (in particular his (1997a) and (2003)) and the mental models approach is less straightforward. Galison's work emphasises the impact that industrial, "hands on" environments have on developments of all kinds in science. In particular, by focussing on the material culture of the scientist's "inner laboratory", i.e. his microenvironment (Galison, 1997a, 4), he aims to show that "the age-old tradition of opposing headwork to handwork, ideas to experiences, and rationalism to empiricism [...] ever more difficult to sustain" (ibid.,46).

While this is certainly true and in full agreement with the mental models approach, Galison's concern are the descriptive contexts that shape approaches of the various practitioners of science and, through these topics, their metaphysical and ontic commitments. My focus, again, has to do with the conditions that make these commitments possible. As such, an analogy between Galison's and my work is not to be placed in a kind of physical/mental-dichotomy but along the following lines: Practices and (frustratingly) real hardware act as modal constraints on possible actions in the "inner laboratory" analogously to the role of material-ladenness of our theoretical concepts as constraints on possible actions in an "innermost laboratory". This is the place where the theoretician bends his axes, surfs the gradient or tickles his space-time points. However, I don't think that attempts to conjecture on the specific "looks" of this laboratory, whether it is, say, a geometric, visualisable lab or subject to temporality, can be very fruitful or are even necessary. We don't need to know how exactly people experience their thoughts to realise that this experience is not captured by classical approaches.

I would like to stress two more points in connection to Galison's work. Firstly, and closely related to the last paragraph: Galison (1997b) analyses the linguistic problems of communicating between experimental and theoretical physicists. To this end, he introduces his notorious trading zone. However, one of the most exciting features of recognising the material-ladenness of theoretical concepts seems to me to be the possibility to embrace this fact and bridge the alleged external/internal divide through deliberately material-laden language.

Secondly, there is an important way in which Galison's work is very valuable for the mental model framework. In my argument, I have traced materiality of Einstein's mental models back to the early roots of pre-scientific thinking. Galison's work, on the contrary, concentrates on the very present environment of his protagonists, the Zeitgeist they practice in. If an adequate mechanism of translation could is found, his findings could be used to investigate how "immediate" effects of material-ladenness are expressed in the mental model framework. This should not be difficult to achieve, given the slot-architecture of the mental models: Inputs that are filled with the output of some old and established mental model may be replaced by radical, new concepts, where the stability of such a replacement crucially only depends on the overall degree of correlation between these two models, etc.

Mental Models and Education

One possible application of this last idea can be distilled from Warwick's (2003) study on the influence of the 19th century pedagogical vogues on the work of wranglers at Cambridge. Warwick's aim is to break down the idea of "theoretical work" as "an essentially cerebral, contemplative, and introspective accomplishment of isolated and outstanding individuals" (Warwick, 2003, 16). Instead, theoretical work is presented and documented as a craft that is to be learned like any other. The notion of craft here incorporates a component of laborious and mistake-ridden study of techniques as well as a material component of working on paper and blackboards.

This is particularly interesting for our present concern in that, in the words of van Dongen (2010, 185), "it is namely natural to conceive of Renn's mental models as the result of intensive training, or some other similarly formative experience". How that? Both the above components have an easy interpretation in the mental models approach. Concerning the first, the moment of inheritance that was discussed in connection with mental models earlier, figures centrally in the master-disciple relationships of didactic contexts. Here, for a novice in a field, the teachings that reflect current academic trends, are his only means to articulate his position in that field and, later on, his criticism. He first has to accept the teachings and learn how to apply them to problems, since he cannot judge their merit himself. This is represented as the coordination of the working flows between different mental models, that is affected by both their global architecture as well as the connection between individual slots. Only after he has succeeded in this task can he grow beyond it. This process, which is referred to by Renn and Sauer (2007, 119) as the "paradox of discontinuous progress", appears in the mental models approach as the successive change of slot parameters of inherited mental models.

The second component, the material influence in formative pedagogical contexts, relates more closely to the main concern of this essay. Here the central question becomes: To what extent does the material conditioning of the pupil affect his cognitive possibilities, what he can and cannot, or at least will or will not, mentally construe or conceive of? The anthropologist Jack Goody discusses the important concept of "material concomitants" of what he refers to as "mental domestication". These concomitants are external representations, "manifestations of thought, invention, creativity"(1977, 9). In mathematics and theoretical physics education such materials they are paper, ink, blackboards, etc., mundane materials that, yet, "can provide an important clue to a practical strategy for analyzing the theoretician's craft in terms of material culture" (Warwick, 2003, 16). What is important to stress is the need for these concomitants as a means of documenting thoughts, progress and communication with peers and teachers. How do these particularly scholarly kinds of materials shape our thinking?

To pick just one example, one might look at two aspects of the way in which paper shapes our mental models and their material-ladenness: For one, there is the question of supply: In times where the for some, possibly political or economical, reason there is very limited availability of paper as the main means of documentation, the resulting practice of studying and researching will be heavily affected: Brainstorming and other interims-work will need to be carried out undocumented, thoughts be communicated orally, etc. It is not difficult to imagine how this mundane fact can heavily influence both the research of mathematicians or theoretical physicists and its results. Secondly, we can think about the shape of paper as the theoretician's canvas: For example, the rectangular shape of the Din A4 format singles out a prominent direction: Top to bottom. This format then quite naturally suggests a linearity of argument with the directed-ness of mental concepts entering tacitly into the mental models. To illustrate this, we may imagine schools in which paper is squared or even circular, i.e. with no prominent direction at all. This change in what one can do with the learning material would be likely to have an effect on the theories that will be learned and improved on it. But this is just another instance of material-ladenness of mental theories, though this time not deriving from the stuff one looks at but from the pedagogical context, i.e. the working material one uses to look at stuff.

We see then how mental models are not only capable of explaining the discontinuous dynamics of progress that happens when students grow to replace their former masters, they also capture the way in which the material culture of education shapes the material-ladenness of our mental models. Importantly, the moral of this analysis of Warwick's book in the light of the mental models approach was to show how the social and economical situation of a single generation influences the development of material-laden mental models alongside long-term influences.

7 Conclusion

To conclude, I have used Howard's analysis of Einstein's separability principle as a case study on the conditions for the ontic treatment of mathematical concepts. The result of this study and the investigation of its implications more generally, is the recognition of the important role that the material-ladenness of mental models (and, hence, the concepts they represent) plays in constituting and constraining the possibility of assigning an ontological function to mathematical concepts. This function is of particular interest in science where questions about the relationship between the mathematical formalism and the physical stuff it represents still await a satisfying answer. In Einstein's case his ontic treatment of the manifold points, upon which the field is mathematically defined, was possible because of the material-ladenness of the mental models of object permanence and impenetrability of matter, where these models underlie the more sophisticated Lorentz model of a field theory that Einstein was working with.

By comparing this presented framework with the work of Chang and Galison we found an important characteristic of the material-laden mental models to be their contingency: Cultural and material influences enter, in Kant's terminology, our "functions of thinking", at least concerning the ontic treatment of mathematical concepts. This influence we can spot on long and short term scales.

Finally, let me emphasise that I intend this paper to be a stimulus rather than a brick. I believe that, by looking at the biographies of our mental concepts in the mental models framework, we can trace some interesting properties of them down to their material-ladenness. These properties, by constraining the applicability of the concepts, shape the working flow of scientists. At the same time, ignorance of the mechanisms that produce these properties may give rise to conceptual problems, both in philosophy and the sciences themselves, that really aren't problems at all. Given that we spend most of our lifetime asking misplaced questions, the study of material-ladenness may allow us to spend more time asking more important misplaced questions instead.

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