INDIVIDUATION IN MODERATE STRUCTURAL REALISM ABOUT SPACETIME

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Abstract. It is desirable, if not crucial, for a philosophical position on the interpretation of general relativity (GR) that commits to the existence of spacetime points to be able to both (1) individuate these points for any model of GR and (2) evade the hole argument of Earman and Norton (1987). Moderate structural realism is a position that makes this commitment. Here, following an introduction of structural realism (Sec.1), I first discuss an argument of Wüthrich (2009) aiming to show that moderate structural realism is incapable of satisfying (1) (Sec.2). I argue that it can be rebutted along the lines presented by Müller (2011) (Sec.3). Then I consider whether the form of moderate structural realism that is required for this rebuttal to work can also manage to satisfy (2), arguing that it can (Sec.4).

1. Introduction

Structural realism (SR), in the philosophy of science, is the position that the claims scientific theories "make about the 'structure' of physical reality are true." (Greaves, 2011, 199). It has been suggested as a 'third way' in the debate between scientific realism and anti-realism, a kind of dialectic synthesis of theirs that evades both the "no-miracle"-argument and the implications of pessimistic meta-induction (Worrall, 1989). Within the *ontic* type - claiming unlike the *epistemic* type not that structure is all we can have *justified belief* in but that structure is all there *is* - four forms of SR are distinguished: Represent a structure[1] set-theoretically as a duple $\mathfrak{S} \equiv \langle \mathfrak{B}, \mathfrak{R} \rangle$, with a "basis" $\mathfrak{B}$ and their relations $\mathfrak{R}$.[2] Then *timid*, *traditional* and *moderate* SR assume that $\mathfrak{S}$ is all there is, in the sense that any world can be fully characterised by giving its structure (possibly non-uniquely), however they assign ontological primacy to $\mathfrak{B}$,$\mathfrak{R}$ and to none of them, respectively. To a fourth, *radical* type, relations are all there is, i.e.

[1]Hoping to avoid unnecessary confusion, I will use the following terminology: At the trans-world-level: "Structures", as defined above, represent "worlds". "Models"(of GR) are isomorphic to structures and therefore also represent worlds. Many structures/models can represent the same world. At the intra-world-level: Elements of the basis of a structure represent elements of the ontology of the world corresponding to that structure. In the case of GR, manifold points are elements of the basis and spacetime points elements of the ontology.

[2]Define the set $\mathfrak{R}^n$ of n-place relations R_n in the standard way as $\mathfrak{R}_n \subseteq \mathcal{P}(\mathfrak{B}^n)$ where $\mathcal{P}$ is the power set and $\mathfrak{B}^n$ is the Nth cartesian power of $\mathfrak{B}$. For a structure with $|\mathfrak{B}| = N$, define $\mathfrak{R} := \bigcup_{n=1}^{N} \mathfrak{R}_n$. The fact that cases with $n = \infty$ are problematic, such as for example the manifolds considered here, is ignored (together with the other authors).

$\mathfrak{S} \equiv \mathfrak{R}$.[3] Moderate structural realism (mSR) is characterised by asserting that there are no intrinsic properties. These are "all and only those qualitative properties whose exemplification is independent of the existence or nonexistence of other contingent objects" Wüthrich (2009, 1041).

In the case of spacetime theories, it has been argued that this exclusion of intrinsic properties allows moderate spacetime structural realism (mSSR) to treat spacetime points ontologically in a way that is weak enough to evade the hole argument of Earman and Norton (1987) but strong enough to not collapse into relationalism (Esfeld and Lam, 2008; Stachel, 2002). However, there are no free lunches, and the question is what price mSSR has to pay for this ability. In the next two sections, I discuss the claim Wüthrich (2009) that the commitment to the existence of spacetime points in mSSR is so weak that there are models of GR for which mSSR is forced into believing that the ontology of these models consists of only a single point at any time. In the last section, I consider whether an mSSR that evades this embarrassment can still solve the hole argument.

2. Too weak to individuate?

In this sections I present Müller's argument by first considering the question of individuation in SR (2.1) and then the argument that builds on it (2.2).

2.1. **Structural individuation.** The elements of the ontology of a theory are usually individuated[4] using a discernability criterion, for example some version of the Principle of the Identity of Indiscernibles (PII).[5] In the case of structural realism about spacetime, in which the basis $\mathfrak{B}$ is given by the manifold $\mathcal{M}$, a property-based[6] PII says that, for a given structure $\mathfrak{S}$ and any $p, q \in \mathcal{M}$,

$$AbsInd(p, q, \mathfrak{S}) \Rightarrow p = q, \tag{1}$$

where $AbsInd(p, q, \mathfrak{S})$ is a function capturing *absolute indiscernibility* and is defined as

$$AbsInd(p, q, \mathfrak{S}) \Leftrightarrow (\forall P \in Prop_{\mathfrak{S}} : p \in P \leftrightarrow q \in P)\,, \tag{2}$$

[3]The distinction between ontic and epistemic SR goes back to Ladyman (1998). Th one between timid, traditional and radical SR is introduced by Stachel (2006), while the moderate version is ascribed to Esfeld and Lam (2008).
[4]I am concerned here only with relations of identity and non-identity, using the term "individuate" as synonymous with "infer a relation of non-identity between".
[5]The PII itself is contentious. See (Saunders, 2003) for some motivations.
[6]A property is just a unary relation, i.e. the set of all properties for a given structures is just $\mathfrak{R}_1$.

where $Prop_{\mathfrak{S}}$ is some set of "discerning properties". Which properties are elements of this set depends of course on the particular philosophical position. In the case of any structural realism, this can include only *qualitative* properties, that is "all and only those properties whose exemplification does not depend upon the existence of any particular individual" (Wüthrich, 2009, 1041). This excludes, for example, *haecceity* - a metaphysical property that lets one individuate a point just by virtue of it being *this* particular point - since this is a non-qualitative property. For the *moderate* version of SR, with its exclusion of intrinsic properties, it must be relational properties, i.e. unary properties of points defined in terms of non-unary relations, for example of the form "$p \in R \in (\mathfrak{R}_1) \Leftrightarrow \forall q : \langle p, q\rangle \in R'(\in \mathfrak{R}_2)$". In fact, these relational properties are exactly those unary properties invariant under actions of the automorphism group $Aut(\mathfrak{S})$ of $\mathfrak{S}$, whose elements ψ map $\mathfrak{S}$ onto itself, leaving $\mathfrak{R}$ invariant. In fact, given the set-theoretic formulation of this essay, each such automorphism is isomorphic to a map $\psi : \mathfrak{B} \to \mathfrak{B}$ with the corresponding group being a subgroup of the permutation group $\Pi(\mathfrak{B})$ over the elements of the basis. In fact, using this map we can represent the action of these automorphisms on subset of $\mathfrak{B}$ and, therefore, any objects of a structure. Define then, for a given structure, $Rel_n \subseteq \mathfrak{R}_n$ as the set of relations that are invariant under such automorphisms of the structure, it then follows that, according to Wüthrice, mSSR is be characterised by

$$Prop_{\mathfrak{S}} \subseteq Rel_1, \tag{3}$$

where we allow for the possibility that not all relational properties can be used to discern.

2.2. **Setting the challenge.** Wüthrich (2009) sets a challenge for mSSR. It bases on the idea that isometries of the metric undermine the latter's individuating capacity and thereby force a follower of mSSR with embarrassingly little means to distinguish between spacetime points.

First take a model of general relativity to be defined by the tuple[7]

$$\mathcal{S} := \langle \mathcal{M}, g, X\rangle, \tag{4}$$

where $\mathcal{M}$ is a differentiable manifold, g is the second rank metric tensor and X represents other structures required to fully specify a solution of GR, such as, for example the stress-energy tensor, the connection and the topology. To each of these models corresponds a unique

[7]I omit any mention of connections etc. here in that they are not required for this discussion and are at least formally derivable from the metric (together with compatibility assumptions). This is *not* intended as a presupposed stance on the ontology of GR.

structure $\mathfrak{S} = \langle \mathfrak{B} \equiv \mathcal{M}, \mathfrak{R} \equiv \mathfrak{R}(\mathcal{S}) \rangle$, in which the basis is given by the manifold[8] and the relations are determined by the model. To be moderate structural realist about GR, as I mean it, is to be moderate structural realist about all the structures corresponding to all the models of GR.

Importantly, this includes those that satisfy the cosmological principle (CP). This principle states that there are no special direction or points in space. This implies that exact spatial homogeneity and isotropy must be global isometries of the metric (invariance under spatial translations and rotations respectively) for any model consistent with the CP. Now, by the Robertson-Walker theorem any such a model will be a FLRW-spacetime (Robertson, 1935). This spacetime, characterised by a FLRW-metric with the above isometries, determines a unique foliation of the manifold $\mathcal{M}$, i.e. its partition into disjoint spacelike hypersurfaces Σ_t, that preserves the spatial symmetries and for which $t \in \mathbb{R}$ can be interpreted as a global "cosmic" time. Call this foliation of $\mathcal{M}$ into the Σ_t's $Cos(\mathcal{S})$.

Next, it is easy to see that, by construction, every Σ_t in this spacetime is invariant under all automorphisms $\psi_t : \Sigma_t \to \Sigma_t$. In fact, it can be shown that

$$Rel_1 \subseteq Cos(\mathcal{S}), . \tag{5}$$

(5) does not follow automatically by the automorphism invariance of the foliation, since the elements of Rel_1 are invariant under automorphisms of the *whole* structure. But this also includes the other elements X as well. Therefore, the maps ψ_t could, in principle, be such that $\psi_t(X) \neq X$ - in the end, the Robertson-Walker result is a geometric result that is independent of the Einstein equation. That (5) is nevertheless true follows by the fact that, CP is imposed as a global constraint even with or without Einstein equations: X *must*, by this principle, be such that the FRLW-metric is a solution to the Einstein equations at all times - a requirement expressed in the Friedmann equation - and *this* means that the a_t's are automorphisms of the whole structure, i.e. that (5) holds.

But these results imply, for Wüthrich, an "abysmal embarrassment" for mSSR:

Take any point $p \in \mathcal{M}$. By (5), any element of Rel_1 that p is a member of is also an element of $Cos(\mathcal{M})$. But because $Cos(\mathcal{M})$ is a partition of $\mathcal{M}$, this implies that p is a member of only one element $\Sigma_t \in Rel_1$ for some t. But by (3), the PII (1) implies that p is identified with

[8] In general, of course, other elements of a model may be taken to form the basis. In this essay, I consider only positions that make the above assignment.

any other point in Σ_t. Thus, the ontology of mSSR in the case of spacetimes obeying the CP consists of a single point at any given time!

3. Meeting the challenge

In this section I discuss a solution to the Wüthrich's argument proposed by Müller (2011). I do so by first introducing it (3.1) and then discussing its feasibility (3.2).

3.1. **Müller's reply.** What reply can the moderate structural realist offer to the "embarrassing" conclusion of the last section? If she (a) does not want to convert to any other form of structural realism and further continues to (b) make a form of the PII the basis of individuating the elements of her ontology as well as (c) consider her ontology to consist of spacetime points rather than, for example, fields, then, Wüthrich (2009, Sec.4) argues, she can only either reject the physical relevance of FLRW- and similarly symmetric spacetimes or consider stronger forms of the PII. The former possibility Wüthrich considers not to be very attractive. This is because even though highly symmetric spacetimes are likely of measure zero, their actual obtaining cannot, at the moment, be excluded. To this one can add that most moderate structural realists will probably share the intuition that homogeneous and isotropic spacetimes consist of many more than a single spacetime point and that following the first option would simply beg the question. Thus, for a moderate structural realist of the above beliefs a strengthening of the PII seems the most reasonable route.

Müller (2011) provides a counter to Wüthrich's challenge along exactly those lines: Wüthrich's characterisation of mSSR neglects, the counter goes, the fact that the moderate structural realist accepts genuine n-ary relations between spacetime points. But there exists a simple binary relation, the causal relation manifested in the light-cone, by means of which all manifold points can be "weakly" discerned. Hence, a characterisation that takes into account mSR's sensitivity to such relations evades the embarrassing consequences of the last section. In more detail, it proceeds as follows:

Müller's begins by arguing that *properties* of manifold points are not the only means that mSSR has to discern them. Just like he is able to discern points via automorphic properties, he is able to discern them using *binary* automorphic relations. Even though Müller himself does not require it for his own argument to succeed, this can, of course, be generalised to an inclusion for general n-ary automorphic relations, something that will prove useful in the last section.

A PII involving such general relations then reads, for a given structure $\mathfrak{S}$ for which $|\mathfrak{B}| = N$,

$$\forall p, q \in \mathcal{M} : \bigwedge_{n=1}^{N} RelInd_n(p, q, \mathfrak{S}, \mathfrak{D}_n) \Rightarrow p = q, \tag{6}$$

where the function $RelInd_n(p, q, \mathfrak{S}, \mathfrak{D}_n)$, *relational indiscernability*, obtains iff

$$\begin{aligned} \forall R \in \mathfrak{D}_n : \langle o, a_1, \ldots, a_{n-1} \rangle \in R &\leftrightarrow \langle o, a_1, \ldots, a_{n-1} \rangle \in R \\ \wedge \langle a_1, o, \ldots, a_{n-1} \rangle \in R &\leftrightarrow \langle a_1, o, \ldots, a_{n-1} \rangle \in R \\ &\vdots \\ \wedge \langle a_1, \ldots, a_{n-1}, o \rangle \in R &\leftrightarrow \langle a_1, \ldots, a_{n-1}, o \rangle \in R. \end{aligned} \tag{7}$$

Here, $\mathfrak{D}_n$ are the n-ary relations that can be used to discern. This function then says that two elements of a basis are relationally indiscernible iff there exists no n-ary discerning relation of which they are not both members in the same place. Clearly, $RelInd_1(p, q, \mathfrak{S}, \mathfrak{D}) = AbsInd(p, q, \mathfrak{S}, \mathfrak{D})$ For the moderate structural realist who can discern by means of automorphic relations Rel_n for the same reason as discussed for relational properties,

$$\mathfrak{D}_n \subseteq Rel_n, \quad \forall n \in \{1, \ldots, N\}, \tag{8}$$

From (8) and (6) then follows that if there exists an n-ary automorphic relation for the FLRW-spacetime with $n \leq N$ then points differing with respect to this relation are discernible. Müller takes the light-cone structure to be exactly such a relation: For any model of GR, let $LC(p)$ be the light cone of point $p \in \mathcal{M}$. Then define the following relation $L(p, q) \in \mathfrak{R}_2$

$$\begin{gathered} L(p, q) \Leftrightarrow \\ (\exists r \in \mathcal{M} : r \in LC(p) \backslash LC(q)) \vee (\exists t \in \mathcal{M} : t \in LC(q) \backslash LC(p)), \end{gathered} \tag{9}$$

i.e. $L(p, q)$ obtains iff the light cones of p and q differ in at least one point. Müller then argues that $\neg L(p, q)$ provides an identity criterion, since by definition

$$\neg L(p, q) \Leftrightarrow \forall r \in \mathcal{M}(r \in LC(p) \leftrightarrow r \in LC(q)) \Leftrightarrow p = q, \tag{10}$$

Not only does this mean that $L(p, q)$ weakly discerns any two different points p and q, (10) can also be used to to show that $L(p, q)$ is automorphic, i.e. $L(p, q) \in Rel_2$. This is because for

$L(p,q)$ to be automorphic is that

$$L(p,q) \Leftrightarrow L(\psi(p),\psi(q)). \tag{11}$$

But this follows by (10) together with the fact that the automorphisms ψ are bijective in that

$$L(p,q) \Leftrightarrow p \neq q \Leftrightarrow \psi(p) \neq \psi(q) \Leftrightarrow L(\psi(p),\psi(q)). \tag{12}$$

Thus, Müller concludes, since it was shown that there exist automorphic relations in any general relativistic spacetime model by means of which all spacetime points can be weakly discerned, including all the points in the hypersurfaces Σ_t's of FLRW-spacetimes, a fully characterised mSSR evades the embarrassing consequences of Wüthrich's argument.[9]

3.2. **Is Müller's reply feasible?** To assess Müller's reply it is important to emphasise that it is the *irreflexivity* of $L(p,q)$ that discerns the manifold points. A binary relation R is irreflexive iff $R(x,x)$ is false for all x in the relation's domain. That is, the reason that points are relationally discernible by it, leaving (7) with $n = 2$ to evaluate as false, is that $L(p,q) \leftrightarrow L(p,p)$ is false whenever $p \neq q$. This form of discernibility is called *weak* discernibility (Saunders, 2003, 293).[10] The two main worries with Müller's reply I therefore take to be the following:

(1) Is the use of non-unary relations in mSSR feasible?
(2) Is the use of irreflexive relations in mSSR feasible?

3.2.1. *Feasibility of non-unary relations.* Wüthrich (2012, p.234) considers the use of discerning relations to be unfeasible in that it involves a question-begging assumption of numerical plurality of points:

> "[t]here is the general worry with [Müller's] resolution, of course, that the assumption of there being an irreflexive relation exemplified in the physical structure at stake means, *eo ipso*, that there are two numerically distinct objects exemplifying the relation. If the point of this resolution was that numerical plurality was to be derived, rather than stipulated, then it seems to fail."

Müller (2011, 1057) thinks this worry is ungrounded:

[9]Since $L(p,q)$ is closely linked to conformal structure, it would be interesting to see whether Müller's account can be re-challenged for the case of conformally symmetric structures.

[10]Weak discernibility is to be contrasted with another, stronger form of relational discernability, *relative* discernibility, for which binary relations obtain in only one order, in which case at least one of the conjuncts of (7) has to be false (ibid.).

> "The aim is not [...] to find out whether there really is more than one spacetime point - for that would, indeed, be circular - but the aim is to find out whether the distinctness of the points can be grounded qualitatively, physically, and structurally, and that has not been assumed tacitly."

What I take Müller to be saying here is that what needs to be shown is that the moderate structural realist has the means of distinguishing between the different points of a general relativistic spacetime *no matter how many of them there are.* This is because it was an asserted inability to distinguish, not an asserted number of points in the universe, on grounds of which the problem arose in the first place. A clear way of seeing this is by clearly distinguishing elements of a basis from those of the ontology: There is nothing question-begging in using bases $\mathfrak{B}$ with any number of points; if they turn to out to all correspond to a single element of the ontology, the PII will tell.

I think that such a reading of Müller answers Wüthrich's worry, however I believe that the former's argument does not actually reflect this agenda. This is because he infers the identity criterion (10) of spacetime points in general relativistic spacetimes from the *definition* of $L(p, q)$. But this is not actually in line with how mSSR goes about reasoning about identity. Instead it is in the *PII* that such a criterion needs to be grounded. Still, Müller's conclusion can be reached with only two more weak assumptions, as I will show now: The aim is to prove that

$$\neg L(p, q) \Leftrightarrow p = q, \tag{13}$$

where we need both directions of this equivalence in order to prove the automorphic invariance of L(p,q) in (12). Now make the following assumption:

$$\begin{aligned} \forall p, q \in \mathcal{M} :&((\exists N \leq n, n \neq 2 : \neg RelInd_N(p, q, \mathcal{M}, \mathfrak{D})) \\ &\vee (\neg RelInd_2(p, q, \mathcal{M}, \mathfrak{D}_2 \backslash L(p, q)))) \\ &\Rightarrow L(p, q) \end{aligned} \tag{14}$$

This says that if there exists a relationally discerning relation for a model that is not $L(p, q)$ such that it obtains for two points $p, q \in \mathcal{M}$, then $L(p, q)$ obtains for these points as well. This assumption is needed because a modus tollens on the PII implies only that there exists *some* relation under which the two points are discernible, not which one. Then the righthand implication follows for absolutely indiscernible manifolds by (14) together with the PII, (6). The lefthand implication follows by (14) together with the Principle of the (absolute and relative)

Indiscernibility of Identicals, i.e.

$$\forall p, q \in \mathcal{M}, \mathfrak{D}_n, \mathfrak{S} : p = q \Rightarrow \bigwedge_{n=1}^{N} RelInd_n(p, q, \mathfrak{S}, \mathfrak{D}_n) \tag{15}$$

The latter assumption is very weak, indeed Müller considers it a theorem of logic (ibid., 1054). The former is considerably stronger but if one is sure that there is some relation other than $L(p, q)$ that discerns two points, then Müller's argument is redundant anyway. By proving in this way, the rest of Müller's argument then goes through and also does justice to Wüthrich's counter.[11]

3.2.2. *Feasibility of irreflexive relations.* An independent question is whether, granting the feasibility of using non-unary relations, the weak discernibility provided by irreflexive properties is admissible in the case of Wüthrich's argument. (Saunders, 2003) defends the applicability of such relations in physics in general, discussing examples such as the sermonic single state. But given that Wüthrich (2009, 1045) uses an irreflexive distance measure to weakly discern to Black's notorious spheres, at least he himself cannot think the notion itself problematic. Any problem he might have with the notion beyond its binarity must therefore be grounded in the particular case of mSSR or even the FLRW-models.

One hint in this direction may be read off a footnote that directly follows the above quote of Wüthrich:

> "To claim that weak discernibility may be used to introduce an identity relation to a language without identity amounts to an insistence that the pertinent relation is *primitively* irreflexive. Thus [. . .] such a claim is tantamount to accepting non-structural facts..." (Wüthrich, 2012, footnote 29, orig. emphasis)

There are two ways in which, I think, Wüthrice may be read here: Either Wüthrich takes the assignment of irreflexivity to $L(p, q)$ as primitively irreflexive to be the only alternative to establishing this irreflexivity on the basis of assuming the numerical plurality of points in the universe. In this case the argument of the last section applies and we need not worry about such an alternative because the original account, to which it was meant to provide an alternative, is not anymore subject to the problem. Or he takes the the irreflexivity of $L(p, q)$ to simply *be* primitive and therefore non-structurally grounded. If this was true, then the irreflexivity of $L(p, q)$ would disqualify it for mSSR. But of course it is not: The irreflexivity of $L(p, q)$

[11]It may be that the alternative argument I sketch below is made tacitly by Müller. However, no mention is made of the PII or any of the other assumptions by him and the fact that he presents this part of the argument as a formal proof suggests that he wouldn't have omitted these details in this case.

follows by (9) set-theoretically as a tautology and is completely determined by the metric, and thus structurally grounded. Thus, it seems that Wüthrich's only point of concern regarding the feasibility of irreflexives was via the use of non-unary relations which they necessarily implied and which was shown in the last section to not be problematic.

Consequently, at least with respect to the two main worries above, Wüthrich's challenge can be tackled by mSSR, albeit in a slightly different form of that presented by Müller himself.

4. Müller's mSSR and the hole argument

So much for the first part of the promise of mSSR, the commitment to the existence of spacetime points. But recall that mSSR is meant to also solve the hole argument. The obvious question to ask in the light of the earlier discussion whether Müller's version of mSSR can solve the problem. If it cannot, the worry goes, then Wüthrich's challenge might remain unanswered after all.

In this section I find the following: An introduction of the hole argument (4.1) makes it clear that Müller's mSSR, as concerned with *intra*-world individuation, cannot solve the hole argument itself but only be *compatible* with a *trans*-worldly individuating mSSR that itself manages to solve the argument (4.2). A tentative version of this latter mSSR is then presented (4.3) and found to be compatible with Müller's stance, indicating that the above worry is ungrounded (4.4).

4.1. **The hole argument.** This argument was originally discussed in (Earman and Norton, 1987). My presentation here bases on a set-theoretic version that Pooley (2006, Sec.6) develops (based itself on (Stachel, 2006)). For any structure $\mathfrak{S} = \langle \mathfrak{B}, \mathfrak{R} \rangle$ that corresponds to a solution of the Einstein equations, the diffeomorphism invariance of these equations implies that any structure $\mathfrak{S}_\Pi = \langle \mathfrak{B}, \mathfrak{R}_\Pi \rangle \equiv \langle \mathfrak{B}_\Pi, \mathfrak{R} \rangle$ that is the result of a permutation $\Pi : \mathfrak{B} \to \mathfrak{B}$ also corresponds to a solution of theirs. Now introduce the possibility of giving descriptions of these structures: Considering a structure $\mathfrak{S}$ with $|\mathfrak{B}| = N$, descriptions are expressed as sentences of first order logic (R are propositions corresponding to relations os the same label and a_k names of elements of $\mathfrak{B}$): Then, assuming for notational convenience that all $\mathfrak{R}_n, n \leq N$ can be reconstructed by element $R \in \mathfrak{R}_N$, a *complete description* of $\mathfrak{S}$, denoted $\mathbb{R}_N(a)$, is a conjunction of three formulas: (i) a conjunction specifying, for every N-place sequence a of the names a_k and every relation $R \in \mathfrak{R}_N$ either $R(a)$ or $\neg R(a)$; (ii) a conjunction $\left(\bigwedge_{i \neq j} a_i \neq a_j\right)$ stating that different names name different elements of $\mathfrak{B}$; (iii) a disjunction $\left(\forall y (\bigvee_{i=1}^{N} y = a_i)\right)$ stating that the a_k are the only elements of $\mathfrak{B}$. In this setup, a *partial* structural description is a formula

$\exists x_1 \ldots \exists x_{(n-m)} \mathbb{R}(b, x_1, \ldots, x_{(n-m)})$ that specifies only a substring $c \equiv (x_1, \ldots, x_{(n-m)})$ of $a \equiv (b, c)$ and is consistent with many different permutationally related structures (corresponding to the full descriptions given by all permutations of the 'hole'-string b).

Importantly, any partial description of a structure is compatible with many complete descriptions that differ by a permutation, all of which are deemed physically possible in that they correspond to models of GR. Consequently, any position that considers structures that differ by a permutation to represent different worlds (such as, allegedly, substantivalism) must accept that any partial specification of a solution to the Einstein equations (specifying, say, the solution only for some time $t \leq t_c$) is compatible with many different possible worlds (i.e. many solutions for $t > t_c$). Hence, the argument concludes, any such position must interpret GR as an indeterministic theory. A position, on the other hand, that identifies all structures related by a permutation to represent only one world, i.e. that produces "Leibniz equivalence", avoids this problem.

4.2. **Inter- and intra-world individuation.** Greaves (2011, 201) points out that mSSR as discussed by either Wüthrich or Müller certainly *cannot* avoid the hole argument in this way. This is because these accounts are concerned with *intra*-world identity of spacetime points, that is, they are discerning points *within* a single world. To establish Leibniz equivalence, on the other hand, an identification of whole structures is required. If identification of manifold points is to be useful here, it can only be via their *trans*-world identification, identity *between* the elements of the bases of different structures.[12] For this a trans-world PII (tPII) is required while both (1) and (6) are intraworld PIIs (iPIIs).

In the light of Greaves' point, the question concerning the ability of Müller's mSSR to solve the hole argument can only be indirectly tackled by asking for a trans-world mSSR that solves the hole argument and is also compatible with the latter, in the following sense: A tPII implies an iPII (but not vice versa)(Greaves, 2011, 202). This is because, if by virtue of a tPII some set of structures $\{\mathfrak{S}_i\}$ is identified and taken to represent the same world, then the set of invertible "transworld" maps $\{\phi_j\}$ - defined by $\exists i, j : \phi_j(\mathfrak{S}_i) = \mathfrak{S}_j$ - induce partitions of the manifold points in the bases $\mathfrak{B}_i$ these structures. The elements of these partitions are formed by all the points $p, q \in \mathfrak{B}_i$ for some structure that can be related by some combination of maps

[12]Greaves further argues that such a transworld mSSR collapses into sophisticated substantivalism, as presented in (Pooley, 2006).This position resolves the hole argument by rejecting *haecceities.* Therefore, whether Greaves' claim is true depends primarily on whether there are non-qualitative properties that are not *haecceities* that are relevant to spacetime theories, in which case the two positions would differ. In any case, this question does not concern us here.

$\phi_j \circ \phi_{j'} \cdots \circ \phi_{j''}(p) = q$, where we used the fact that any ϕ_j can be expressed as a mapping between elements of bases in $\{\mathfrak{S}_i\}$, just as in the case of the automorphisms ψ. Also note that the $\{\phi_j\}$ form a group, which we call $Trans(\{\mathfrak{S}_i\})$. Now, the iPII that is implied by a tPII that identifies the elements of this set then is just the iPII that identifies all members in these elements of the partitions. Calling then any iPII that is implied by a tPII "compatible" with it, to look for a trans-world mSSR that is compatible with Müller's mSSR therefore is to look for a tPII that induces (6) with $\mathfrak{D}_n = Rel_n$ in the way just described.

4.3. **A tPPI for mSSR.** Curiously, no explicit statement of a tPII for mSSR seems to be exist in the literature, at least to the author's knowledge. Instead, the step from relational invariance to identity is simply taken for granted. Consider, for example, the following quote from (Esfeld and Lam, 2008):

> "[The hole argument] can hence be avoided by claiming that *there is no physical individuation within GR of space-time points independently of the metric.* Indeed, this can be seen as the moral of the [...] invariance under certain manifold point transformations taking one point into another together with the induced drag-along maps acting on the tensor fields. Therefore, [...] two diffeomorphic space-time models of the theory should be considered as representing the same physical situation (solution) and the same physically possible space-time structure. This is often referred to as Leibniz equivalence in the philosophical literature about space-time. " (ibid., 36, orig. emph.)

The "certain manifold point transformations" alluded to are diffeomorphisms, the points in question here are most likely manifold points of *different* structures and the authors slide to transworld identification. However, it is left unspecified how such an identification should give rise to Leibniz equivalence.[13]

With no help from the literature, we can nevertheless try to formulate a feasible tPII ourselves: Two objects A and B are defined to be *transworld* identical, $A =_t B$, iff there is some possible world w_1, and some distinct possible world w_2, such that A exists in w_1, and B exists in w_2, and A is identical with B (Mackie and Jago, 2013). In the case of mSR, what will have to ground this is, in Stachel's words, that elements are individuated "by their position in the relational structure" (Stachel, 2002, 236).

[13]Note, further, that the authors consider the relational structure to be fully specified by the metric, in constrast to the more general accounts discussed here.

I take it that a notion of *transworld relational indiscernability* based on Stachel's words can be captured by the function $TransRelInd_n(o, o', \mathfrak{B}, \mathfrak{B}', \mathfrak{R})$ that obtains iff, for two structures with the same $\mathfrak{R}$ and $|\mathfrak{B}| = |\mathfrak{B}'| = N$, for any $n \in \{1, \dots, N\}$

(1) $o, a_1, \dots, a_n \in \mathfrak{B} \wedge o', a_1', \dots, a_n' \in \mathfrak{B}'$

(2) $\forall k \in \{1, \dots, n\} : \quad a_k =_t a_k'$

(3)

$$\begin{aligned}\forall R \in \mathfrak{R}_n : (&\langle o, a_1, \dots, a_{n-1}\rangle \in R \leftrightarrow \langle o', a_1', \dots, a_{n-1}'\rangle \in R\\ &\wedge\langle a_1, o, \dots, a_{n-1}\rangle \in R \leftrightarrow \langle a_1', o', \dots, a_{n-1}'\rangle \in R\\ &\vdots\\ &\wedge\langle a_1, \dots, a_{n-1}, o\rangle \in R \leftrightarrow \langle a_1', \dots, a_{n-1}', o'\rangle \in R).\end{aligned}$$

This is of course reminiscent of $RelInd(p, q, \mathfrak{S}, \mathfrak{D}_n)$. The important difference, manifest in the absence of the paramter $\mathfrak{D}_n$ is that $TransRelInd$ is *already* a statement of mSR, namely in requiring that two transworldly relationally indiscernible points are indiscernible with respect to *all* elements of the same $\mathfrak{R}$.

The corresponding tPII, then states that for any two points $o \in \mathfrak{B}, o' \in \mathfrak{B}'$

$$(\exists \mathfrak{S} = \langle \mathfrak{B}, \mathfrak{R}\rangle, \mathfrak{S}' = \langle \mathfrak{B}', \mathfrak{R}\rangle : TransRelInd(o, o', \mathfrak{B}, \mathfrak{B}', \mathfrak{R})) \Rightarrow o =_t o'. \tag{16}$$

Finally, using (16), two structures $\mathfrak{S}$ and $\mathfrak{S}'$ are then identified by mSR iff all elements of their ontologies are transworld identicalfootnoteOf course, the requirement 2 in the definition of $TransRelInd$ means that evaluating the identity of two structures using this tPII will be impossible because it is circular. The argument here requires, however, only a theoretical criterion for the identity of structures., i.e.

$$\mathfrak{S} = \mathfrak{S}' \Leftrightarrow o =_t o' \quad \forall o \in \mathfrak{B}, o' \in \mathfrak{B}'. \tag{17}$$

Clearly, by (17), $\mathfrak{S} = \mathfrak{S}_\Pi$. In this way, (16) yields a tPII that solves the general hole argument of Sec.4.1 and, in particular, establishes Leibniz equivalence for the case of GR in which $\mathfrak{S}$ and $\mathfrak{S}_\Pi$ correspond to diffeomorphically related solutions of the Einstein equations.

4.4. **Establishing compatibility.** Finally, we are now in a position to answer the main question of this section: Is Müller's mSSR compatible with a transworld mSSR that solves the hole argument? Recall from Sec.4.2, that this question boils down to asking whether (16) implies (6) with $\mathfrak{D}_n = Rel_n$. And here the answer is clearly affirmative: (16) expresses the requirement that structures be identified if they share the same relations. But this invariance of the relational structure was just what restricted the moderate structural realist to be able to intra-worldly individuate elements of the basis using only *automorphically* invariant properties. In terms of the maps ϕ_j, the automorphism group for a single structure is exactly the group that is induced by the transworld-group in that, for every $\psi_j \in Aut(\mathfrak{S}_i), \quad \exists \phi_j, \phi_i \in Trans(\{\mathfrak{S}\}) : (\phi_i \circ \phi_j)(\mathfrak{S}_i) \equiv \psi_j(\mathfrak{S}_i)$.

Thus, if (16) gives a feasible tPII for a moderate structural realist position, then Müller's solution to Wüthrich's challenge is, I conclude, successful.

Conclusion

Summing up, in this essay I discussed the ability of moderate structural realism about spacetime to commit to the existence of spacetime points while at the same time standing a chance to solve the hole argument. In particular, I argued that a mSSR that transworldly identifies worlds via (17) implies an iPII (6) that successfully individuates spacetime points even in highly symmetric models of GR along the lines of Müller (2011).

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